Roll No. ....

94063

B. Sc. (Hons.) (Mathematics) 5th Semester Examination – March, 2021

METHODS OF APPLIED MATHEMATICS

Paper : BHM-355

Time : Three Hours] *[Maximum Marks : 60*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting *one* question from each Unit. Question No. 9 is *compulsory*. All questions carry equal marks.

UNIT – I

1. (a) To find the displacement of the vibrating string of length L whose ends are fixed. Given that $f(x)$ is initial displacement and initial velocity is zero.

   (b) Solve three dimensional wave equation in cylindrical polar coordinates.

2. (a) A string of length L is initially at rest in its equilibrium position and each of its points is given the velocity $\left(\frac{\partial u}{\partial t}\right)_{t=0} = 40\sin^3\left(\frac{\pi x}{L}\right)$ where $0 < x < L$, find the displacement function $u(x,t)$.

   (b) Obtain the solution of Laplac's equation in cylindrical polar coordinates.

UNIT – II

3. (a) Write the solution for :

   $$\frac{\partial u}{\partial t} = 4\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \text{ for } 0 < x < 2,\ 0 < y < 3,\quad t > 0$$

   with conditions that :

   $$u(x,0,t) = u(x,3,t) = 0 \text{ for } 0 < x < 2,\ t > 0$$

   $$u(0,y,t) = u(2,y,t) = 0 \text{ for } 0 < y < 3,\ t > 0$$

   and $u(x,y,0) = x^2(2-x)\sin(y)(3-y)$

   (b) Solve the problem $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$ for $-\infty < x < \infty, t > 0$ and $\mu(x,0) = e^{-4|x|}$ for $-\infty < x < \infty$.

4. (a) Solve wave equation for semi-infinite string.

   (b) Solve the following B. V. P. :

   $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} = \frac{1}{a^2}\frac{\partial u}{\partial t}$ with boundary conditions $u(b,t) = 0$ and $u(r,0) - f(r)$.

UNIT – III

5. (a) Find the nth order Hankel Transform of $f(r) = r^n H(a - r)$.

(b) Find the solution of the Laplace equation for the steady temperature distribution $u(r, z)$ with a steady and symmetric heat source $Q_0\, q(r)$.

6. (a) Find the Fourier cosine transform of e^{-x^2}.

(b) Using the Fourier sine transform, solve the partial differential equation $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$ with the boundary condition

(i) $u = u_0$ when $x = 0,\ t > 0$ and the initial condition.

(ii) $u = 0$ when $t = 0,\ x > 0$.

UNIT – IV

7. (a) State and prove parallel axes theorem.

(b) Find the M. I. of a rigid circular solid cone of mass M, height h and radius a of base about its axis.

8. (a) Prove that principal axis are mutually orthogonal.

(b) Show that two systems are equimomental iff :

(i) they have same total mass,

(ii) they have same centroid,

(iii) they have same principal moments of inertia at centroid and same principal axis.

UNIT – V

9. (a) A square of side 'a' has particle of masses m, 2m, 3m, 4m at its vertices. Find the M. I. and P. I. at the centre of square.

(b) Write solution of $P^2R'' + PR' + m^2P^2R = 0$.

(c) Write Laplace's equation in spherical polar co-ordinates.

(d) State perpendicular axis theorem.

(e) Obtain the zero order Hankel Transforms $r^{-1}\exp(-ar)$.

(f) Find the Fourier cosine transform of $2e^{-5x}$.